

Debt Constraints and Unemployment

Virgiliu Midrigan

New York University

Patrick J. Kehoe

University of Minnesota

and University College London

Elena Pastorino

University of Minnesota

and University College London

May 2014

ABSTRACT

We develop a matching model with risk averse consumers, borrowing constraints, and upward sloping wage-tenure profiles and investigate its ability to account for some key features of the Great Contraction of the United States. This contraction was associated with a large fall in household debt to income, consumption, and employment. Moreover, as several authors have noted, in the most recent recession, regions of the United States that experienced the largest swing in household borrowing also experienced the largest drops in output and employment. Finally, in the regions that experienced this large drop in employment, the drop was disproportionately large in the nontraded goods sector. We show that our model can reproduce the salient features of this contraction.

We develop a Diamond-Mortenson-Pissarides model with risk averse consumers, borrowing constraints, and upward sloping wage-tenure profiles and investigate its ability to account for some key features of the Great Contraction of the United States. This contraction was associated with a large fall in household debt to income, consumption, and employment. Moreover, as Mian Sufi (2009) and Midrigan and Phillopon (2011) have noted, in the most recent recession, regions of the United States that experienced the largest swing in household borrowing also experienced the largest drops in output and employment. Finally, in the regions that experienced this large drop in employment, the drop was disproportionately large in the nontraded goods sector. We show that our model can reproduce the salient features of this contraction.

A key feature we add to that model is human capital acquisition through employment, so as to generate realistic wage-tenure profiles of the type documented by Buchinsky, Fougère, Kramarz, and Tchernis (2010). The key idea is that faced with such upward-sloping patterns of wages with tenure, an unemployed consumer who is close to the borrowing constraint realizes that accepting a job means accepting a path of consumption that tracks these wages. Moreover, since the firm shares the surplus from the match with the consumer the firm also faces a path of profits that is backloaded as well. Either an unanticipated tightening of borrowing constraints or an unanticipated fall in the value of consumers' assets, say, from a fall in house prices, together with a binding collateral constraint makes the equilibrium discount rate used by consumers and firms increase. Thus such curved paths with backloaded returns are less attractive to both consumers and firms and this increase in the discount rate makes the surplus from a match fall.

The main reaction to this tightening comes from the firm side. After witnessing such unanticipated changes, firms think through the outcome of their bargaining with consumers once they post a costly vacancy and meet with a consumer. Specifically, firms realize that since both their returns and the consumer's returns are backloaded then after such changes, the present value of profits from posting a vacancy fall. Hence, in equilibrium firms decrease the number of vacancies they post and unemployment rises whereas output falls. This pattern continues through a lengthy transition: as long as the borrowing constraint binds, both consumers and firms discount the future highly and find jobs offering the prospect of increasing

wage profiles relatively less attractive. An interesting feature of this transition is that the tightening generates a path of increased unemployment that lingers and seemingly ‘sticky’ wages, despite wages being continually renegotiated, as consumers slowly adjust their asset positions given the tighter borrowing constraints.

As emphasized by Hall (2013), the key unresolved aspect of the Mortensen and Pissarides model is that nature of the force that depresses the payoff to job creation in recessions. In most of the papers in this vein, the key force is a drop in productivity. As Hall also emphasizes, that explanation for the recent period runs into two problems. First, productivity did not fall much in the Great Contraction. Second, given that small fall in productivity, it is exceedingly difficult for the model to generate a large drop in output and employment. The driving force that we emphasize is an unexpected tightening of credit constraints on consumers.

As noted, one key feature we added to the KMS framework is human capital acquisition during employment. A second key feature is financial frictions on the consumer side imply that when the borrowing constraint tightens, consumers discount the future relatively highly.

Our work is related to a small literature that tries to link increases in financial frictions on the consumer side to regional economic downturns. In particular, Guerrieri and Lorenzoni (2010), Eggertson and Krugman (2011), and Midrigan and Phillipon (2011) study macroeconomic responses to a household-side credit crunch. All three of these papers find that a credit crunch has only a minor effect on employment if the economy is away from a so-called zero lower bound.

1. The Economy

We consider a version of the Mortenson Pissarides model in which consumers accumulate firm specific human capital on the job and their productivity in the market increases with experience. We assume that consumers can insure against idiosyncratic shocks but are subject to collateral constraints.¹ As we show, with human capital accumulation,

¹We also solved a version of the model similar to that in Krusell et al. (2010) in which consumers can only save with uncontingent bonds. This model has an unappealing feature. Because of human capital accumulation, the wage that solves the Nash bargaining problem is nonmonotonic in individual assets. In particular, there is a region of the parameter space in which higher assets reduce wages. Anticipating this some consumers are deterred from savings because by doing so it decreases their wages. This features led to

the equilibrium interaction between the tightness of borrowing constraints and consumer and firm behavior is critical for our results.

The economy consists of a continuum of consumers of measure 1. In any period, a consumer is either matched with a firm or is unemployed and searching for a match. We begin by setting up and defining a stationary equilibrium for our economy using recursive notation, in which primes denote next period variables. We assume a standard aggregate matching function $M(u, v)$, which represents the measure of matches in a period with u unemployed workers and v vacancies. We assume M is homogenous of degree 1 and initially let $M(u, v) = Bu^\eta v^{1-\eta}$. The probability that a vacant job is filled in the current period is $\lambda_f(\theta) = M(1/\theta, 1) = M(u, v)/v$ where $\theta = v/u$ is the vacancy-unemployment ratio, whereas the probability that an unemployed worker finds a match is $\lambda_w(\theta) = \theta\lambda_f(\theta) = M(u, v)/u$.

A. Consumers

Consumers, often referred to as workers, differ in their productivity in the workplace denoted z_t . A worker with human capital or *productivity* z_t produces

$$y_t = z_t$$

when employed at time t and, regardless of z_t , produces b unit of home production when unemployed. When the consumer is employed, productivity evolves according to

$$(1) \quad \log z_{t+1} = (1 - \rho)\bar{z}_e + \rho \log z_t + \sigma_\varepsilon \varepsilon_{t+1},$$

where ε_{t+1} is distributed normally with mean zero and variance 1. When the consumer is unemployed, the value of z_t for the consumer evolves according to

$$(2) \quad \log z_{t+1} = (1 - \rho)\bar{z}_u + \rho \log z_t + \sigma_\varepsilon \varepsilon_{t+1}.$$

We will assume $\bar{z}_u < \bar{z}_e$. We represent the Markov processes in (1) and (2) as $F_e(z_{t+1}|z_t)$ and $F_u(z_{t+1}|z_t)$ in what follows.

odd patterns of wages, savings, and sometimes even the existence of equilibrium.

We interpret z_t as capturing the gains in productivity through human capital accumulation through tenure and experience in the labor market. When the consumer is employed, on average, the variable z_t drifts up toward a high level of productivity, \bar{z}_e . When the consumer is unemployed, on average, the variable z_t depreciates slowly toward a low level of productivity, \bar{z}_u which we normalize to 1. Note that when a consumer is unemployed, the consumer produces a constant amount of goods b in home production and z_t represents what the consumer would be able to produce if employed. By setting appropriately the parameters of the two processes for z_t , we can match gains from tenure and experience as well as the loss in wages a consumer experiences upon separation. (Here we do not formally distinguish between unemployment and nonparticipation but when quantifying the model, we will think of non-employment as capturing both states.)

Each consumer survives from one period to the next with probability ϕ and a measure $1 - \phi$ of new consumers are born in each period. Hence, there is always a measure 1 of consumers. Newborn consumers start as unemployed with $\log z$ drawn from $N(\bar{z}_u, \sigma_z^2/(1-\rho_z^2))$.

We represent the insurance arrangements in the economy by imagining that each consumer belongs to a family which has a continuum of household members each of which experiences idiosyncratic shocks. The family as a whole receives a deterministic amount of income in each period from the income generated by its working and non-working members. Risk-sharing within the family implies that, in period t each household member consumes the same amount of consumption goods, c_t and housing services h_t regardless of the idiosyncratic shocks that such a member experiences. (This type of risk sharing arrangement is familiar from the work of Merz (1995) and Andolfatto (1996).) The family as a whole is subject to time-varying collateral constraints that prohibit the family from borrowing more than a fraction of the value of the house the family owns.

Given this setup we can split the problem of the family into two parts: a consumption allocation problem for the common amount of consumption in the family and the employment allocations for each member of the family. Consider the consumption allocation problem for the family given by

$$\max_{c_t, h_{t+1}, a_{t+1}} \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(h_t)]$$

subject to a budget constraint

$$(3) \quad c_t + q_t a_{t+1} + \rho_t h_{t+1} = a_t + \rho_t h_t + y_t + T_t$$

and a collateral constraint

$$(4) \quad q_t a_{t+1} \geq -\chi_t \rho_t h_{t+1}.$$

Here the effective discount factor β is the product of the discount factor $\tilde{\beta}$ and the survival probability ϕ , y_t represents the deterministic income of the family as a whole, q_t represents the world bond price, and ρ_t is the price of one unit a house. Here $\{\chi_t\}$ is an exogenous deterministic sequence of maximal loan to value ratios, so that a consumer with a house of value $\rho_t h_{t+1}$ can borrow no more than a fraction χ_t of this value. Finally, we assume firms are owned by the household and let T_t be the profits from all firms net of the vacancy posting costs.

Our model of houses is very simple. There is a fixed supply of houses, normalized to 1, and each unit of a house delivers one unit of housing services each period, and there is no depreciation. Here buying h_{t+1} units of a house at t entitles the owner to a stream of housing services of h_{t+1} in all periods from $t + 1$ on.

We assume the economy is small and open and can borrow from the rest of the world at a constant rate $q_t = \bar{q}$. We assume that $\beta < \bar{q}$ so that consumers in our economy are impatient relative to those in the rest of the world. In this economy the family will value one unit of goods at time t in unit of goods in period 0 according to $Q_{0,t} = \beta^t u'(c_t)/u'(c_0)$.

Consider now the second part of the family problem: the employment allocations for each member of the family. Clearly, the optimal employment decisions for the family can be attained by instructing each household member to choose employment so as to maximize the present value of output using the family level discount factor $Q_{0,t}$ for goods in period t . That

is, if $y(s^t)$ represents the stream of income, including goods produced in home production, generated by an individual member of the family, this member chooses employment so as to maximize

$$\max \sum_{t=0}^{\infty} \sum_{s^t} Q_{0,t} y_t(s^t)$$

where we have focused on the problem of a consumer born at date zero. Note that from (7)

The problem of a newborn consumer is similar except that the income stream for a consumer born at t is zero before t . Here, for ease of notation, we have represented the insurance arrangements using a family construct. It should also be clear that we can also represent these insurance arrangements by having appropriately defined contingent claims markets.

We can now write the problem of the consumer recursively. We will posit and then later characterize the equilibrium wages as the outcome of a certain bargaining problem described below that yields $w = \omega_t(h, z)$. For a given wage w and productivity level z the present value of income of an employed consumer is

$$(5) \quad \tilde{W}_t(w, z) = w + Q_{t+1}(1 - \sigma) \int \max[W_{t+1}(z'), U_{t+1}(z')] dF_e(z'|z)$$

$$(6) \quad + Q_{t+1}\sigma \int U_{t+1}(z') dF_e(z'|z)$$

where the one-period ahead discount factor is $Q_{t+1} = Q_{0,t+1}/Q_{0,t} = \beta u'(c_{t+1})/u'(c_t)$ and $W_t(z)$ is defined as $\tilde{W}_t(\omega_t(z), z)$. Notice that first order condition for consumption can be written

$$(7) \quad \beta \frac{u'(c_{t+1})}{u'(c_t)} = q_t - \frac{\theta_t}{u'(c_t)}$$

where $\beta^t \theta_t$ is the normalized multiplier on the borrowing constraint. Hence if the collateral constraint binds at t then the consumer will discount the future returns from matching at a rate higher than the world rate q_t . Notice also that (5) allows the consumer to endogenously separate from the firm when the productivity shock z' is sufficiently low. Clearly there is an optimal endogenous separation rule: separate from the match if $z \leq \bar{z}_t$. The value of an

unemployed consumer is

$$(8) \quad U_t(z) = b + Q_{t+1}\lambda_{w,t} \int \max [W_{t+1}(z'), U_{t+1}(z')] dF_u(z'|z)$$

$$(9) \quad + Q_{t+1}(1 - \lambda_{w,t}) \int U_{t+1}(z') dF_u(z'|z).$$

Note that in both (5) and (8) we have written the present value of income starting from period t in units of period t goods, in particular $Q_{t+1} = Q_{0,t+1}/Q_{0,t}$ is the price of consumption at $t + 1$ in units of date t consumption goods.

B. Firms

Each firm acts as an independent profits center and forms matches with consumer, produces output, and pay dividends to the family. The objective of each firm is to maximize the discounted value of dividends using the discount factor $Q_{0,t}$ of the family

Each firm has one job that can either be vacant or filled. In any period a firm with an unfilled job can pay a fixed cost κ to create a vacancy and attract a worker at the beginning of the next period with probability $\lambda_f(\theta)$. The value of a filled vacancy for a consumer with productivity z is

$$(10) \quad \tilde{J}_t(w, z) = z - w + Q_{t+1}(1 - \sigma) \int \max (J_{t+1}(z'), 0) dF_e(z'|z)$$

where $J_t(z) = \tilde{J}_t(\omega_t(z), z)$. Note that in (10) we have allowed the firm and the worker to voluntarily separate if after matching the stochastic productivity falls sufficiently so that value of a continued match turns negative. Clearly there is an optimal endogenous separation rule: separate from the match if $z \leq \bar{z}_t$. As we show below, since the Nash bargaining solution for wages implies that consumer's surplus is positive if and only if firm's surplus is positive and hence the cutoff rule for endogenous separation rule for consumers and firms are the same. Next, the value of an unfilled vacancy is

$$V = -\kappa + Q_{t+1}\lambda_{f,t} \int_{z'} \int_z \max [J_{t+1}(z'), 0] dF_e(z'|z) d\tilde{n}_t^u(z)$$

where $\tilde{n}_t^u(z)$ is the distribution of unemployed workers with $\tilde{z} \leq z$, that is the measure of

unemployed workers with $\tilde{z} \leq z$, namely $n_t^u(z)$, scaled by the measure of all unemployed workers. In equilibrium, free entry into vacancy creation implies that $V = 0$.

C. Bargaining and Equilibrium

Wages are set each period by Nash bargaining where

$$\left[\tilde{W}_t(w, z) - U_t(z) \right]^\gamma \left[\tilde{J}_t(w, h, z) - V \right]^{1-\gamma}.$$

Noting that $\partial \tilde{W}_t / \partial w = 1$ and $\partial \tilde{J}_t / \partial w = -1$ the wage $w = \omega_t(h, z)$ is given by

$$\frac{\gamma}{\tilde{W}_t(w, h, z) - U_t(h, z)} = \frac{1 - \gamma}{\tilde{J}_t(w, z)}$$

where we have used the free entry condition.

In equilibrium the income of the household is

$$(11) \quad y_t = \int \omega_t(z) dn_t^e(z) + b \int dn_t^u(z) + T_t$$

where $n_t^e(z)$ is the measure of unemployed workers with $\tilde{z} \leq z$.

2. Quantification and Results

The model is quarterly. The discount factor $\tilde{\beta}$ is $(.94)^{1/4}$, the world bond price $\bar{q} = (.96)^{1/4}$, the survival rate ϕ is set so that $1 - \phi = 1/160$ so that households are in the market for 40 years on average. The probability of separation $\sigma = .1$ is set so that the average employment spell is about 2 and 1/2 years as in Shimer (2005). The bargaining weight γ is set to 1/2 and we set the elasticity of the matching function $\eta = \gamma$. The average productivity level of the unemployed \bar{z}_u is normalized to 1. The utility function is

$$u(c_t) = \frac{c_t^{1-\alpha}}{1-\alpha}$$

We set $\alpha = 5$ in line with work in the asset pricing literature. with $\alpha = .5$. The persistence of the productivity shock $\rho = (.95)^{1/4}$ and the standard deviation is $\sigma_\varepsilon = 2.5\%$ from Floden and Linde (2001). The parameters: b , the production of the unemployed, \bar{z}_e , the mean

productivity of the employed, B , the efficiency parameter in the matching function and the parameters λ and \bar{h} of firm specific human capital accumulation were set to minimize the distance between four moments in the model and targets in the data: a ratio of income for the unemployed to the employed of 65%, an employment to population ratio of 80% which is about the rate for 24 to 60 year olds in the United States, a drop in wages of 15% on average for a consumer that separates for one period and then is rematched and a drop in wages of 20% on average for a worker that separates and then is rematched in 5 periods and that wages grow on average at 10% per year when employed. A large literature (see Guvenen 2006 and the references therein) finds that the elasticity of intertemporal substitution (EIS) is very low (on the order of 0.1 to 0.2) when estimated using data on households. We follow this literature and set this elasticity, $1/\alpha$, equal to 0.2.

We have 6 additional parameters that are jointly chosen so that the model matches exactly 6 statistics in the data. These parameters are: κ , the fixed cost of posting a vacancy; B , the efficiency of the matching function; ρ_z , the persistence of productivity shocks, σ_z , the standard deviation of productivity shocks, μ_z , the parameter governing the evolution of z for an employed worker, and b , the home production parameter. The targets we use to pin down these moments are: i) an employment-population ratio of 80%, ii) the mean growth rate of wages of 5.2% implied by the Buchinsky et. al (2010) estimates of returns to tenure and experience, iii) a steady-state vacancy to unemployment ratio of 1 following Shimer 2005, iv) a ratio of home production to mean wages of 40%, following Shimer 2005, v) the standard deviation of the log of initial wages of 0.94, which we compute using the PSID data, and vi) the standard deviation of changes in log wages of 0.21 per year, as computed by Floden and Linden 2001.

Intuitively, some parameters in the model have relatively more importance for some statistics in the data. Roughly, the fixed cost of posting vacancies pins down the steady state vacancy-to-unemployment ration, the efficiency of the matching function pins down how often non-employed agents are matched with the firm and thus pins down the employment-population ratio, the parameter governing the law of motion of z for employed workers, μ_z , pins down the growth rate of wages, while the persistence and standard deviations of shocks to z determines the unconditional dispersion and the volatility of changes in wages. As Table

1 shows, the model matches all 6 parameters exactly given that the model is exactly identified.

A. Steady State

Consider first the steady state. Since consumers in our economy are impatient relative to the world bond price, the collateral constraint binds in a deterministic steady state. We then consider experiments in which the maximal loan to value ratio falls so that the collateral constraint binds along the path to a new steady state. Hence, along this path $q_t a_{t+1} = \chi_t \rho_t$ where we have substituted that $h_{t+1} = 1$ in equilibrium.

The two graphs in the left column under decision rules show that when a firm hires a worker the profits $z - \omega(z)$ are negative for a range of productivities z . The bottom left panel shows that the value of a newly filled vacancy $J(z)$ is zero when z is approximately equal 1 (FIX). As z increases the value of a newly filled vacancy increases. Comparing the top and the bottom left hand panels show that a firm creates matches with consumers even for levels of productivity in which profits are initially negative. The reason is that over time z tends to grow and leads future profits to be positive. In this sense, there is an investment aspect to the relationship between firms and consumers in a match in addition to the cost of posting the vacancy.

The graph on the top right shows the value $\omega(z) - b$ for different values of z . As the graph shows, an unemployed consumer accepts a job below the current value of home production b , so that “worker profits” are negative. The consumer finds it optimal to do so because of the potential for future positive values of $\omega(z) - b$ the will occur as the worker accumulates both productivity z . The graph on the bottom right shows when z is near 1 (FIX) a consumer is indifferent between accepting a job and continuing to search. For higher values of productivity the worker strictly prefers the job. Notice that here also, in contrast to standard search models, there is also an investment component to the consumer from forming a match with the firm.

The next page of graphs shows features of the ergodic steady state. The top graph shows that about 8% of consumers have the lowest level of human capital, namely 1. These consumers are the sum of newly born workers and non-newly born workers that are unemployed. The bottom two graphs shows the marginal measures $G_e(z)$ and $G_u(z)$ of employed

and unemployed consumers by productivity level z . That the lower support of employed workers is near 1 reflects the fact that firms only consumate matches with consumers with z in that range.

The ergodic distribution of the unemployed shows a sizeable mass of consumers with z near one. Many of these consumers are new borns that start with productivity z , equal to 1 others are non-newborn unemployed with z 's near the mean of the unemployed. Since the firm will not hire unemployed workers with z much less than 1 the mass to the left of $z = 1$ tends to build up.

B. A Tightening of Credit

We now turn to our experiment. In it we suppose that the credit limit is gradually tightened over several quarters so that consumption drops 5% in the first couple of quarters.

In the first period this tightening is unexpected and after that consumers have perfect foresight. This tightening causes the interest rate to rise. Thus, relative to the initial steady state with an lower interest rate, the consumer discounts the future increase in wages that a job implies at a higher rate. In the next set of figures we show how the firm and consumer decision rules change from those before the tightening (at a given market tightness θ). The tightening of credit markets makes a given upward sloping profile of wages less attractive to a consumer because the consumer discounts the future by more. That force occurs because as the credit constraints tighten consumption falls and the marginal utility rises, especially in the near term. In equilibrium, consumers would need to be compensated with higher wages to accept a job. Firms realize this and hence cut back on vacancies. Hence, when consumers in existing matches lose their jobs, they spend a longer time in unemployment since firms have cut back their demand for workers. This effect leads to a prolonged downturn in employment and a fall in output.

3. An economy with traded and nontraded goods

We consider an that consists of a continuum of islands each of which produces a nontraded good for its island and a distinct variety of traded goods. This continuum of islands taken together is a small open economy relative to the rest of the world. There is a fixed stock of houses of size 1 on each island. The available insurance arrangements are captured by

having consumers on each island belong to a representative family that insures island-specific idiosyncratic risks but the family as a whole is subject to island-specific fluctuations in the collateral constraint. A representative family on each island owns the firms on this island. Each island can borrow and lend from the world economy at a world bond price $q_t = \bar{q}$ subject to collateral constraints. Each island faces a downward-sloping demand curve for the variety of traded goods that it produces.

A. Consumers

Let $j \in [0, 1]$ index the islands. Our family construct ensures that every currently living family member in period t consumes the same amount of traded goods $c_{Tt}(j)$, nontraded goods $c_{Nt}(j)$, and houses $h_t(j)$. The preferences of the family, which are an aggregate of the individual member's preferences, are then given by

$$(12) \quad \sum_{t=0}^{\infty} \beta^t [u(c_t(j)) + v(h_t(j))]$$

where the composite good

$$c_t(j) = \left[\omega^{\frac{1}{\mu}} (c_{Nt}(j))^{\frac{\mu-1}{\mu}} + (1-\omega)^{\frac{1}{\mu}} (c_{Tt}(j))^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}.$$

where $\omega^{\frac{1}{\mu}} + (1-\omega)^{\frac{1}{\mu}}$ is normalized to 1. The traded good is a composite of the varieties of other traded goods produced on a continuum on the other islands. So

$$c_{Tt}(j) = \left(\int c_{Tt}(j, j')^{\frac{\nu-1}{\nu}} dj' \right)^{\frac{\nu}{\nu-1}}$$

where $c_{Tt}(j, j')$ are purchases by agents on island j of variety of the traded good produces on island j' . Letting $p_{Tt}(j')$ denote the price of imports of island j' , the demand for an individual variety j'

$$c_{Tt}(j, j') = \left(\frac{p_{Tt}(j')}{P_{Tt}} \right)^{-\nu} c_{Tt}(j)$$

where the aggregate price for traded goods P_{Tt} is the same for all islands

$$P_{Tt} = \left(\int p_{Tt}(j')^{1-\nu} dj' \right)^{\frac{1}{1-\nu}}$$

We assume the each unemployed consumer can produce b units of the composite good. Hence, the total amount of aggregate consumption good produced by unemployed agents $b_t(j)$ is b times the measure of the unemployed agents.

Let $p_{Nt}(j)$ be the price of nontradable goods on island j . The demand for the nontradables on island j is

$$c_{Nt}(j) = \omega \left(\frac{p_{Nt}(j)}{P_t(j)} \right)^{-\mu} (c_t(j) - \bar{b}_t(j))$$

where the price of the consumption bundle of nontraded and the composite traded goods on island j is

$$P_t(j) = [\omega (p_{Nt}(j))^{1-\mu} + (1-\omega) (P_{Tt})^{1-\mu}]^{\frac{1}{1-\mu}}$$

The demand for tradables produced by island j come from two sources: the sum of all the island's demands plus the demand from the rest of the world. This total demand for *exports* from island j is given by

Here the c^{ROW} must also include anything that uses it, say their fixed costs as well.

$$\begin{aligned} & (1-\omega) \left(\frac{p_{Tt}(j)}{P_{Tt}} \right)^{-\nu} \int \left(\frac{P_{Tt}}{P_t(j')} \right)^{-\mu} (c_t(j') - \bar{b}_t(j') + v_{Nt}(j') + v_{Tt}(j')) dj' \\ & + \left(\frac{p_{Tt}(j)}{P_{Tt}} \right)^{-\nu} c_t^{row} \end{aligned}$$

where c_t^{row} denotes the demand from the rest of the world for the differentiated traded good from country j from all sources. We assume c_t^{row} is an exogenously given sequence. We will focus on an experiment in which we tighten the credit constraint on one island only. That tightening will lead this island to change its production of nontraded and traded goods, and thus affect both price of island j nontraded good $p_{Nt}(j)$ and the island j traded good $p_{Tt}(j)$. Since the composite traded good is made up a continuum of goods, this tightening will have

no effect on the composite traded goods price, P_{Tt} , but, of course, it will effect the price of the consumption bundle on island j , namely $P_t(j)$.

On the production side, there are firms producing nontradables and firms producing tradables. All firms are competitive. On island j the nontradable firms sell their output at $p_{Nt}(j)$ while the tradable firms sell their output at $p_{Tt}(j)$.

The family problem for island j has two parts. The consumption allocation problem for a given island is to maximize (12) subject to the budget constraint

$$(13) \quad P_t(j)c_t(j) + q_t a_{t+1}(j) + \rho_t(j)h_{t+1} = a_t(j) + \rho_t(j)h_t + P_t(j)y_t(j) + T_t(j)$$

and a collateral constraint

$$(14) \quad q_t a_{t+1}(j) \geq -\chi_t(j)\rho_t(j)h_{t+1}.$$

Here we express all prices at t in units of the composite tradable good (which here is equivalent to setting $P_{Tt} = 1$). In particular, in (13) the world bond is a promise to pay a certain amount of composite traded goods. For this economy we assume as before that the world bond price is $q_t = \bar{q}$ and that $\beta = \tilde{\beta}\sigma < \bar{q}$ so that consumers on this continuum of islands are impatient relative to those in the rest of the world. Also, in the budget constraint the household labor income and income from ownership of firms are expressed in units of the composite traded goods. In this economy the family will value one unit of composite traded goods in period t in units of composite traded goods in period 0 according to

$$Q_{0,t}(j) = \frac{\beta^t u'(c_t(j)) P_0(j)}{u'(c_0(j)) P_t(j)}.$$

(Note that aggregate consumption goods are a composite of nontraded goods and composite traded goods.)

The second part of the family problem is to determine the employment allocations for each member of the family on this island. Here the optimal employment decisions for the family can be attained by instructing each household member to choose employment so as

to maximize the present value of income measured in units of the composite good by using the family level discount factor $Q_{0,t}(j)$ for composite goods in period t . Let s^t denote the idiosyncratic shock history for a individual consumer and $y(j, s^t)$ represents the stream of income in units of composite traded goods, including goods produced in home production, generated by that consumer. The individual consumer chooses employment so as to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} Q_{0,t}(j) y_t(j, s^t)$$

B. Matching Technology

On any island, an unemployed worker simultaneously searches for a match in both sectors. If a worker with z units of human capital finds a match in the nontraded goods sector produces z units of nontraded goods in that sector while if that worker finds a match in the traded goods sector the worker produces z units of the differentiated good j that is produced by the island j . From now on we will focus on a particular island, say island j , so we find it notationally convenient to drop the j index. Let

$$u_t = \int dn_{ut}(z)$$

be the measure of unemployed on the island. (Newborn agents all start in the unemployment pool.) Let v_{Tt} be the number of vacancies posted in the traded sector and v_{Nt} be the number of vacancies posted in the nontraded sector on that island. The number of matches in the traded sector and nontraded sectors are then

$$M_{Tt} = B_T (u_t)^\eta (v_{Tt})^{1-\eta}$$

and

$$M_{Nt} = B_N (u_t)^\eta (v_{Nt})^{1-\eta}$$

where we allow for the efficiency of the matching function to be sector-specific.

We make assumptions that ensure that a worker is matched in any given period with a

single firm: either firm in the traded good sector or one in the nontraded goods sector. Given that M_{Tt} matches are formed in the tradable good sector and M_{Nt} matches are formed in the nontraded goods sector, the probability that a worker is matched with a firm in the traded goods sector is

$$\lambda_{T,t}^w = \frac{M_{Tt} + M_{Nt}}{u_t} \frac{M_{Tt}}{M_{Tt} + M_{Nt}} = \frac{M_{Tt}}{u_t} = B_T \left(\frac{v_{Tt}}{u_t} \right)^{1-\eta} = B_T (\theta_{Tt})^{1-\eta}$$

This says that since there $M_{Tt} + M_{Nt}$ total matches created, the worker has a $(M_{Tt} + M_{Nt}) / u_t$ chance of being matched. Multiplying that by the fraction of the matches that are in the traded goods sector, $M_{Tt} / (M_{Tt} + M_{Nt})$, gives the overall probability of being matched with a firm that has posted a vacancy in the traded goods sector. Likewise

$$\lambda_{N,t}^w = \frac{M_{Tt} + M_{Nt}}{u_t} \frac{M_{Nt}}{M_{Tt} + M_{Nt}} = \frac{M_{Nt}}{u_t} = B_N \left(\frac{v_{Nt}}{u_t} \right)^{1-\eta} = B_N (\theta_{Nt})^{1-\eta}$$

Consider next the firm's job finding probabilities. Since there are M_{Tt} total matches created and v_{Tt} vacancies, the job-filling probability in the traded goods sector is

$$\lambda_{T,t}^f = \frac{M_{Tt}}{v_{Tt}} = B_T \left(\frac{v_{Tt}}{u_t} \right)^{-\eta} = B_T (\theta_{Tt})^{-\eta}$$

and the job filling probability in the nontraded goods sector is

$$\lambda_{N,t}^f = \frac{M_{Nt}}{v_{Nt}} = B_N \left(\frac{v_{Nt}}{u_t} \right)^{-\eta} = B_N (\theta_{Nt})^{-\eta}$$

C. An Individual Consumer's Problem

An individual consumer with human capital z can be matched with a job in the tradable sector, matched with a job in the nontradable sector or unemployed. The value of a consumer with a match in the traded sector is

$$\begin{aligned} W_{Tt}(z) &= \omega_{Tt}(z) + Q_{t+1} (1 - \sigma) \int \max [W_{Tt+1}(z'), U_{t+1}(z')] dF_e(z'|z) \\ &\quad + Q_{t+1} \sigma \int U_{t+1}(z') dF_e(z'|z) \end{aligned}$$

and the value of a consumer with a match in the nontraded sector is

$$W_{Nt}(z) = \omega_{Nt}(z) + Q_{t+1}(1 - \sigma) \int \max[W_{Nt+1}(z'), U_{t+1}(z')] dF_e(z'|z) \\ + Q_{t+1}\sigma \int U_{t+1}(z') dF_e(z'|z).$$

while the value of an unemployed worker is

$$U_t(z) = P_t b + Q_{t+1}\lambda_{T,t}^w \int \max[W_{Tt+1}(z'), U_{t+1}(z')] dF_u(z'|z) \\ + Q_{t+1}\lambda_{N,t}^w \int \max[W_{Nt+1}(z'), U_{t+1}(z')] dF_u(z'|z) \\ + Q_{t+1}(1 - \lambda_{Tw,t} - \lambda_{w,t}^n) \int U_{t+1}(z') dF_u(z'|z)$$

D. Firm's problem

There are two types of firms, those that produce nontradables with price p_{Nt} , and island-specific tradables with price p_{Tt} . The technology is the same as earlier: a worker with m productivity z produces z units of the nontradable good if matched in the nontradable sector and z units of the tradable good if matched in the tradable goods sector.

The value of a matched firm in the tradable sector is thus:

$$J_{Tt}(z) = p_{Tt}z - \omega_{Tt}(z) + Q_{t+1}(1 - \sigma) \int \max(J_{Tt+1}(z'), 0) dF_e(z'|z)$$

and that in the nontradable sector is:

$$J_{Nt}(z) = p_{Nt}z - \omega_{Nt}(z) + Q_{t+1}(1 - \sigma) \int \max(J_{Nt+1}(z'), 0) dF_e(z'|z)$$

The free-entry conditions into the traded sector is

$$0 = -P_t \kappa_T + Q_{t+1}\lambda_{T,t}^f \int \max[J_{Tt+1}(z'), 0] dF_u(z'|z) d\tilde{n}_{ut}(z)$$

while in the nontraded sector it is

$$0 = -P_t \kappa_N + Q_{t+1} \lambda_{N,t}^f \int \max [J_{Nt+1}(1, z'), 0] dF_u(z'|z) d\tilde{n}_{ut}(z)$$

where $d\tilde{n}_{ut}(z) = dn_{ut}(z) / \int dn_{ut}(z)$ is the probability of meeting an unemployed worker with productivity less than or equal to z .

Notice we allow for the possibility that the vacancy-posting cost differs across sectors.

E. Wage Bargaining

In the traded goods sector we have

$$\omega_T(z) = \arg \max_{\omega} [W_{Tt}(z) - U_t(z)]^{\gamma} J_{Tt}(z)^{1-\gamma}$$

Since we assume period by period renegotiation,

$$\frac{\partial W_{Tt}(z)}{\partial \omega_T(z)} = 1 \text{ and } \frac{\partial J_{Tt}(z)}{\partial \omega_T(z)} = -1$$

and therefore we have that wage in that sector satisfies:

$$\frac{\gamma}{W_{Tt}(z) - U_t(z)} = (1 - \gamma) \frac{1}{J_T(z)}$$

F. Steady State

We normalize prices in the initial steady state so that the steady state of the model is identical to that in the one-good model. We set $P_t = 1$ to be the numeraire. (this is the economy-wide price – we assume all islands are identical in the initial steady state).

To replicate the steady-state of the one-good model, we make the following assumptions on technology. First, we assume costs of posting vacancies, κ^x and κ^n , vary across sectors: we choose them so that the equilibrium market tightness is equal across sectors and equal to 1 ($\theta^x = \theta^n = 1$). Of course, the choice of 1 is a normalization. Second, we would like the wages in the two sectors to be equalized, so that worker's are indifferent between which sector to go to, which happens only if $p_t^n = p_t^x = 1$ (the last equality due to our

normalization of $P_t = 1$). To ensure the former equality holds we need to assume that the matching technology parameters differ across sectors, $B_T \neq B_N$. We choose one of these parameters to match the steady-state employment rate of 80%, and the second one to ensure the nontradable market clears. Let

$$q_t^n = \int hz dG_t^{e,n}(h, z)$$

be the production of nontradables. Let \bar{b} be the total amount of the composite good produced by the unemployed on the island:

$$\bar{b}_t = b \int dG_t^u(z) = b(1 - e_t)$$

Let

$$y_t^n = \int \omega_t^n(h, z) dG_t^{e,n}(h, z)$$

be the total wage income of those in the nontradable sector, and

$$y_t^x = \int \omega_T(z) dG_t^{e,x}(h, z)$$

be the total wage income of those in the tradable sector. Let

$$y_t = y_t^x + y_t^n + P_t \bar{b}$$

be total income.

Consider now the budget constraint of the household in the initial steady state in which all islands are identical. We have

$$p_t^n c_t^n + p_t^x c_t^x + (1 + r) d_{t-1} = d_t + y_t^n + y_t^x + P_t \bar{b}$$

We assume the debt limit is

$$d_t = \phi y_t$$

so equilibrium consumption in steady state is

$$\bar{P}\bar{c} = y_t(1 - \phi r)$$

The market clearing for nontradables (where we use that fact that $c_t - \bar{b}_t$ is the total amount of consumption that must be bought on the market, the rest is home-produced) is

$$\omega \left(\frac{p_t^n}{P_t} \right)^{-\mu} (c_t - b_t) = q_t^n$$

Given that we normalize $P_t = 1$ in the symmetric steady state, we do not need to also impose the market clearing for the nontradable sector. But we will need to use it later on, so we note that tradables are consumed by households on all islands as well as by the risk-free financial intermediaries/firm owners. So we have

$$(1 - \omega) \left(\frac{p_t^x}{\bar{P}^m} \right)^{-\nu} \left(\frac{\bar{P}^m}{\bar{P}} \right)^{-\mu} (\bar{c} - \bar{b}) + \left(\frac{p_t^x}{\bar{P}^m} \right)^{-\nu} \bar{c}^I = q_t^x$$

where c^I are the intermediaries's total use of the tradable good (both consumption and vacancy-posting costs) and $\left(\frac{p_t^x}{\bar{P}^m} \right)^{-\nu} \bar{c}^I$ is the amount they purchase from an island with an export price p_t^x .

The intermediaries consume, in steady state, all the profits from owning firms and interest, less the cost of posting vacancies, so

$$\bar{P}^m \bar{c}^I = r\phi\bar{y} + \bar{p}^n \bar{q}^n - \bar{y}^n + \bar{p}^x \bar{q}^x - \bar{y}^x$$

(note we do not need to separately calculation the fraction of \bar{c}^I used for consumption and that used in vacancy-posting costs since financial intermed. are risk neutral and their consumption does not affect allocations. Moreover, since the vacancy posting costs are denominated in

units of the composite tradable good, their price is constant at $\bar{P}^m = 1$).

4. References

- Andolfatto, David. 1996. Business Cycles and Labor-Market Search. *American Economic Review*, 86 (1)1112-132.
- Chari, V.V., P. Kehoe, and E. McGrattan. 2005. Sudden Stops and Output Drops. *American Economic Review*, 95(2), 381-387.
- Eggertson, G. and P. Krugman. 2011. Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach. Princeton University mimeo.
- Guerrieri, V, and G. Lorenzoni. 2010. Credit Crises, Precautionary Savings, and the Liquidity Trap. Chicago Booth mimeo.
- Hall, R.E. 2013. High Discounts and High Unemployment. Hoover Institution mimeo.
- Krusell, P., T. Mukoyama, and A. Sahin. 2010. Labor-Market Matching with Precautionary Savings and Aggregate Fluctuations, *Review of Economic Studies*, 77(4), 1477-1507.
- Mendoza, E. 2010. Sudden Stops, Financial Crises, and Leverage. *American Economic Review*, 100(5), 1941-1946.
- Merz, Monika. 1995. Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36 (2), 269–300.
- Mian, A., and A. Sufi. 2009. The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis. *Quarterly Journal of Economics*, 124(4), 1449-1496.
- Midrigan, V., and P. Thomas. 2011. Household Leverage and the Recession. NYU mimeo.